

# Arithmetic Progressions (A.P.)

## Definition

List of numbers in which each term is obtained by adding a fixed number to the preceding term except the first term, is called arithmetic progression.

## Common Difference

Fixed number in arithmetic progression is common difference ( $d$ ) which provides the to and fro terms by adding/ subtracting from the present number. Common difference can be positive or negative or zero.

## General form

$$a, a + d, a + 2d, a + 3d, \dots, a + (n - 1)d$$

## $n$ th term i.e., $a_n$

### From end

$$a_n = l - (n - 1)d$$

Here,  $l$  is last term  
 $d$  is common difference  
 $a_n$  is  $n$ th term from end

### From beginning

$$a_n = a + (n - 1)d$$

Here,  $a$  is first term  
 $d$  is common difference  
 $a_n$  is  $n$ th term from beginning

## Example

How many 2-digit numbers are divisible by 7?

2-digit numbers divisible by 7 are 14, 21, 28, ... 98

$$a = 14, d = 7, a_n = 98$$

$$a_n = a + (n - 1)d$$

$$98 = 14 + (n - 1) \times 7$$

$$\text{i.e., } n - 1 = \frac{84}{7} = 12$$

$$n = 13$$

## Arithmetic mean

If  $a, b, c$ , are in A.P.,

$$b = \frac{(a + c)}{2}$$

$b$  is arithmetic mean

## Sum (S)

When first term and common difference are given :

$$S_n = \frac{n}{2} (2a + (n - 1)d)$$

$a$  is first term  
 $d$  is common difference  
 $n$  is total terms

When first & last term are given :

$$S_n = \frac{n}{2} (a + a_n) \text{ OR } S_n = \frac{n}{2} (a + l)$$

$a$  is first term  
 $n$  is total terms  
 $a_n$  is  $n$ th term  
 $l$  is last term

Sum of first  $n$  positive integers

$$\text{Let } S_n = 1 + 2 + 3 + \dots + n$$

$$a = 1, \text{ last term } l = n$$

$$S_n = \frac{n(a + l)}{2} = \frac{n(1 + n)}{2}$$